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LOW VOLTAGE FREE ELECTRON LASER

VOLUME II - THEORY

29 June 1981

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800 N. Quincy Street
Arlington, VA 22217

Prepared by:

W. J. Schafer Associates, Inc.
10 Lakeside Office Park
Wakefield, MA 01880

W J SCHAFER ASSOCIATES INC • 10 LAKESIDE OFFICE PARK • WAKEFIELD MASSACHUSETTS 01880 • (617) 246-0450

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Physical Sciences Division
Office of Naval Research
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Naval Research Laboratory
Attn: Code 2627
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Bldg. 5 Cameron Station
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The theory of the low voltage free electron laser has been surveyed. The physics of trapping and high extraction are discussed. The technical issues connected with the two stage FEL optics are discussed.

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1.0 INTRODUCTION

W. J. Schafer Associates, Inc. (WJSA) has been studying the implications of the availability of the Free Electron Laser (FEL) on strategic military applications, as well as tactical applications under DARPA and Navy contracts[1-6]. These studies were directed towards the conversion of highly relativistic electron beam energy into laser radiation in the 0.3μ-10μ wavelength range using magnetostatic undulators. In this report, we discuss the physics and technical issues of the low voltage FEL ($\lesssim$ 10MeV) wherein the magnetostatic wiggler is replaced by electromagnetic radiation as a pump at the appropriate wavelength for up conversion into visible/near IR radiation. In a separate classified report, we discuss some of the applications of interest to the Navy[7].

In Section II, we present a survey of the theory of the low voltage free electron laser. Various gain formulae are presented in the different operating regimes. These are conveniently mapped in a parameter space chosen so that one can read off the gain from the nomograph for any given set of parameters. In Section III, we discuss the physics of trapping the electrons in the ponderomotive potential well formed by the beating of the pump and scattered wave fields. In the variable parameter magnetostatic wigglers, the trapped electrons are slowed down by decelerating the potential well by means of appropriate variation of the period and field strength of the undulator; with an electromagnetic pump, this is not effective since the period cannot be changed and the field strength change caused by focussing or defocussing of the pump radiation changes the resonance condition only minimally. An alternative suggestion for increasing the energy extraction has been to re-accelerate the electrons with an axial electric field to keep them trapped. Mathematically, these two different concepts have been shown to be identical[8]. We have examined the re-acceleration scheme. We have found that the extraction efficiency and available power depend critically on the emittance of the electron beam. We find, further, that the losses in the optical cavity of the pump beam must be $\leq 10^{-4}$ per round trip if the overall system efficiency is to be respectable.

In Section IV, we discuss the problem of the low voltage FEL optics. Because of the electromagnetic pump, two optical cavities are needed for the operation of the FEL: one at the pump wavelength and one at the upshifted output wavelength. Both of these cavities have to be high Q cavities, especially the one at the pump wavelength, if one aims for good overall system efficiency. Section V summarizes our present understanding and conclusions of the low voltage FEL.

SECTION 2

2.0 SURVEY OF FEL THEORY

2.1 Introduction

Even though interest in the free electron laser has increased tremendously since the theoretical work by Madey[9] and the experimental demonstration of gain and oscillation by Madey and his co-workers[10,11], the history of the free electron laser predates Madey's work by almost twenty years[12]. Chief among the early workers are Pantell, Soncini and Putnoff[13] who, in 1968 proposed a standing electromagnetic microwave pump for up-conversion into the near IR and visible wavelengths. Much of the recent work has been reported in the two proceedings of summer workshop held at Telluride, Colorado[14,15].

Because the value of γ (total electron energy in units of rest energy) treated here is comparatively low ($\lesssim$ 30), a low voltage FEL should be operated in a multistage configuration if a wiggler magnetic field is used as a pump to produce visible radiation. Such a scheme has been proposed by Elias[16] and others[17]. The output of the first stage FEL in such a device is used as the pump for the second stage and so on. As we shall see presently, it becomes quickly impractical to have more than two stages for the generation of visible radiation. The first stage of the FEL could, however, in principle, be replaced by an efficient infrared laser like the CO_2 laser operating at 10μ to produce visible laser radiation in a single stage. The frequency up-conversion with an electromagnetic pump is approximately $4\gamma^2$. The results of the different investigations can be best summarized by the work of Kroll and McMullin[18] who have used the Boltzmann-Vlasov equation in deriving the small signal gain of the FEL in various regimes. These results, therefore, contain both the single particle limit and collective effects obtained by other investigators. Their work is summarized in Section 2.2. This section consists of discussions of fundamental approximations used and the dependance of gain on the system's parameters.

In Section 2.3, the fundamental approximations used in Section 2.2 will be examined in detail. For high voltage FEL, the approximations commonly used seem to be satisfactory. Nevertheless, those approximations valid in a high voltage FEL are seen to be rather limited for a low voltage FEL, not only because of low γ, but also due to high electron current. A discussion of other work in the FEL theory is given in Section 2.4.

2.2 Various Gain Formulae

The gain Γ to be given below is proportional to k_o, the wavevector associated with the pump. Define

$$g = \Gamma/k_o \tag{2.1}$$

This dimensionless quantity g depends on the following dimensionless parameters: (a) γ, (b) the pumping parameter P,

$$P = \left(\frac{eB_o}{mc^2 k_o}\right)^2 \tag{2.2}$$

(c) the dimensionless density parameter Q,

$$Q = \left(\frac{\omega_p}{k_o c}\right)^2 \tag{2.3}$$

where ω_p is the beam plasma frequency,

$$\omega_p = (4\pi ne^2/m)^{\frac{1}{2}} \tag{2.4}$$

and (d) the fractional width S in the electron momentum distribution function $f_o(p)$, normalized according to

$$\int dp\, f_o(p) = 1 \tag{2.5}$$

In terms of electron current density J, ω_p can be written as

$$\omega_p\ (\text{sec}^{-1}) = 8.2 \times 10^{10}\ \beta^{-\frac{1}{2}} \left(\frac{J}{10^4\ \text{amp/cm}^2}\right)^{\frac{1}{2}} \tag{2.6}$$

where

$$\beta = v/c \tag{2.7}$$

In deriving the peak gain formula, Kroll[18] made the following two fundamental assumptions:

(A) The scalar potential produced by electrons in the beam is treated as a self-consistent field. The collective effect obtained is entirely due to this mean field approximation. The collision integrals in the Boltzmann equation are not included.

(B) The radius of an electron helical orbit $r_\perp$,

$$r_\perp = \frac{e\, B_o}{mc^2 k_o^2 \beta\gamma} \tag{2.8}$$

satisfies the condition

$$k_o r_\perp << 1 \tag{2.9}$$

This is called one-dimensional approximation by neglecting the transverse gradient of the wiggler. In terms of P and γ , condition (2.9) can be re-expressed as,

$$\sqrt{P/\gamma} << 1 \tag{2.10}$$

since $\beta \simeq 1$ also for low voltage FEL. For a high voltage FEL, this is well satisfied even for $P \simeq 1$, but this is not so when $\gamma \sim 1$ as in low voltage FEL.[19]

Based on the above approximations, one easily derives the dispersion relation (k_s, ω_s) for the scattered radiation,

$$\left(\frac{\omega_s}{c}\right)^2 - k_s^2 = \frac{1}{\gamma}\left(\frac{\omega_p}{c}\right)^2 - \left(\frac{k}{\gamma}\right)^2 P \frac{\chi(k,\omega)}{1+\chi(k,\omega)} \tag{2.11}$$

where

$$k = k_o + k_s \tag{2.12}$$

with $\omega = \omega_s$ for a wiggler pump and $\omega = \omega_s - \omega_o$ for the electromagnetic pump. $\chi(k,\omega)$ is the electron susceptibility,

$$\chi(k,\omega) = -\frac{m\omega_p^2}{k} \int dp \frac{f_o'(p)}{kv - \omega} \tag{2.13}$$

From Equation (2.11), Kroll derived the following peak gain formulae for g defined in Equation (2.1):

(A) Single Particle Limit (Compton Regime)

$$g = \frac{\pi}{4}\left(\frac{P}{\gamma 2}\right) Q\, S^{-2} \quad \gamma^{-1} \tag{2.14}$$

subject to either

$$P << \tfrac{1}{2}\sqrt{\frac{Q}{\gamma 3}} << S \tag{2.14a}$$

or

$$\tfrac{1}{2}\sqrt{\frac{Q}{\gamma^3}} \ll P \ll S \tag{2.14b}$$

or

$$\tfrac{1}{2}\sqrt{\frac{Q}{\gamma^3}} \ll S \ll P, \quad P\frac{Q}{\gamma^3} \ll 4S^3 \tag{2.14c}$$

(B) Conventional traveling wave tube theory

$$g = 2.18\ (P/\gamma^2)^{\frac{1}{2}} \quad Q^{2/3} \quad \gamma^{-1/3} \tag{2.15}$$

subject to either

$$\tfrac{1}{2}\sqrt{\frac{Q}{\gamma^3}} \ll S \ll P, \quad S^3 \ll \tfrac{1}{4}Q\frac{P}{\gamma^3} \tag{2.15a}$$

or

$$S \ll \tfrac{1}{2}\sqrt{\frac{Q}{\gamma^3}} \ll P \tag{2.15b}$$

(C) Collective Effect Limit (Raman Regime)

$$g = \left(\frac{P}{\gamma^2}\right)^{\frac{1}{2}} \quad Q^{1/4} \quad \gamma^{1/4} \tag{2.16}$$

subject to either

$$P \ll S \ll \tfrac{1}{2}\sqrt{\frac{Q}{\gamma^3}} \tag{2.16a}$$

or

$$S \ll P \ll \tfrac{1}{2}\sqrt{\frac{Q}{\gamma^3}} \tag{2.16b}$$

As mentioned earlier, the validity of Equations (2.14) - (2.16) is subject to the condition (2.10). Since low voltage FEL is likely to be operated in the collective regime, Equations (2.15) and (2.16) show that enhancement due to small γ is rather insignificant if P is scaled as γ^2 because of (2.10). Even if a low voltage FEL is operated in the single-particle limit, it is not obvious that g can be made much larger than the case of high voltage FEL, again because of condition (2.10).

Equations (2.14) to (2.16) are conveniently mapped in a single nomograph in Figure 1 in the parameter space of P, Q and S. For a given set of parameters, one traces through Figures 1(a), (b), (c) and (d) to find the normalized gain. An example is indicated in the figure.

Before going to examine the physical implication of the condition (2.10) for low voltage FEL, it is perhaps worthwhile to derive an expression for the electron current J in the transition from single-particle limit to collective regime. In the present mean field approximation, according to Equation (2.11), the important factor is

$$\chi(k,\omega)/\left[1 + \chi(k,\omega)\right] \tag{2.17}$$

In general, $\chi(k,\omega) << 1$, and for this case, the gain is small. To get high gain, we must make the denominator in (2.17) small. This is limited by the fact that $\chi(k,\omega)$ is complex, but clearly we do best by making

$$\mathrm{Re}\left[1 + \chi(k,\omega)\right] = 0 \tag{2.18}$$

This is the condition to be used to determine the current necessary for the collective effect to be important.

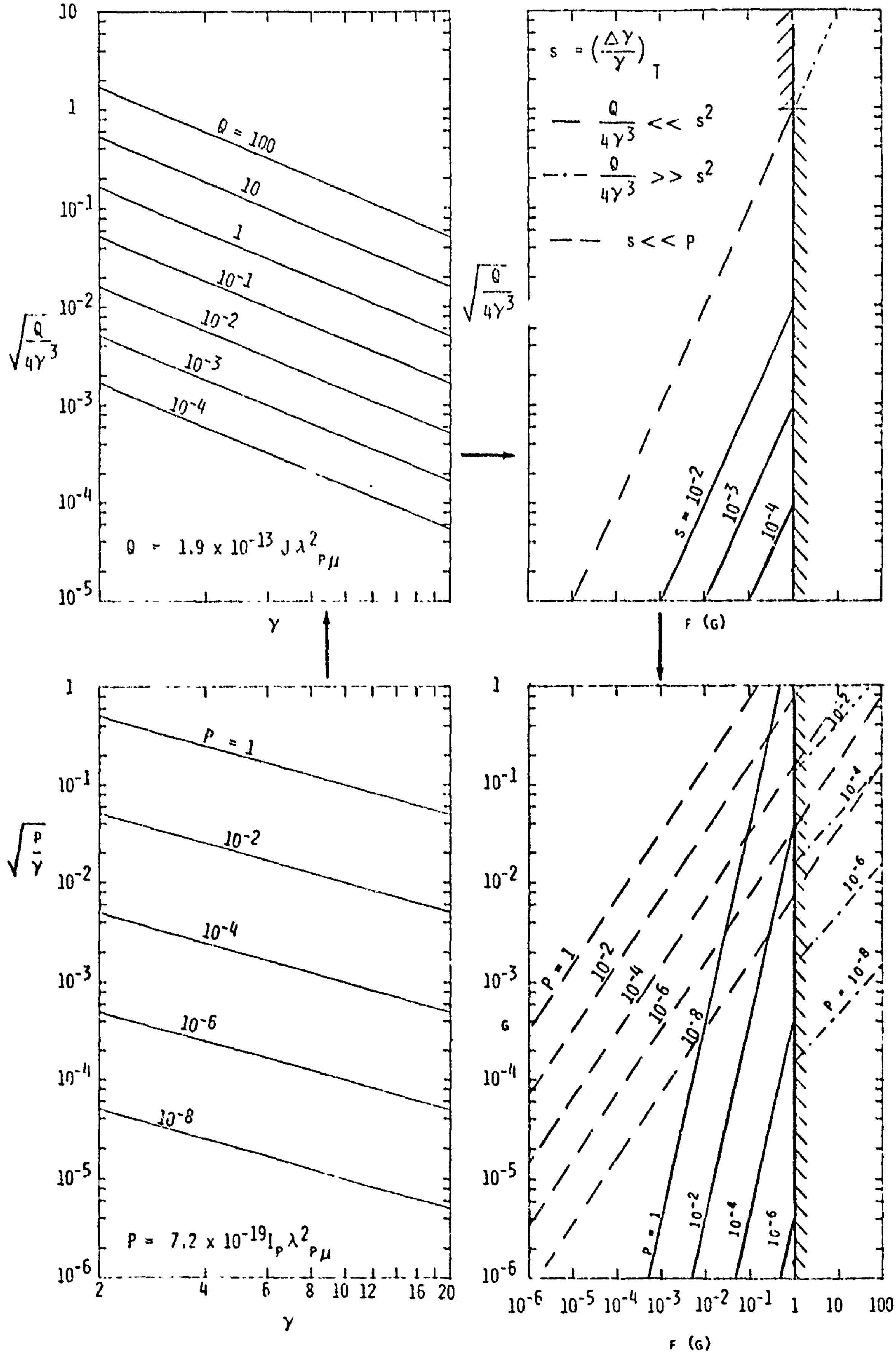

Figure 1. Nomogram to calculate Small Signal Gain

Let p_1 be the momentum which makes the denominator in Equation (2.13) equal to zero,

$$v(p_1) = \omega/k$$

Then $\chi(k,\omega)$ can be rewritten as

$$\chi(k,\omega) = -\frac{m^2\omega_p^2\gamma^3}{k^2}\int dp\ \frac{f_o'(p)}{p-p_1} \tag{2.19}$$

The quantity $v(p)$ is nearly real since k has a very small imaginary part. The real part of χ is then, in excellent approximation,

$$\mathrm{Re}\ \chi(q) = -\frac{m^2\omega_p^2\gamma^3}{k^2}\ P\int dp\ \frac{f_o'(p)}{p-q} \tag{2.20}$$

where P denotes the principal value and

$$q = \mathrm{Re}\ p_1 \tag{2.21}$$

The value of Re χ depends sensitively on q. There is a maximum of $-\mathrm{Re}\ \chi(q)$; whether this is as large as 1 depends, of course, on ω_p^2.

We take f_o to be the Maxwell distribution,

$$f_o = \frac{1}{\sqrt{2\pi}\ \Delta p}\ e^{-x^2/2}$$

$$x = (p - p_o)/\Delta p \tag{2.22}$$

$$\Delta p = \gamma\sqrt{mk_BT}$$

where γ in Δp comes from the Lorentz transformation. Inserting into Equation (2.20),

$$\mathrm{Re}\ X(q) = \frac{m^2\ \omega_p^2\ \gamma^3}{\sqrt{2\pi}\ k^2\ (\Delta p)^2}\ P\int dx\ \frac{x}{x-y}\ e^{-x^2/2} \tag{2.23}$$

$$y = (q - p_o)/\Delta p$$

Letting

$$v_{th}^2 = k_B T/m$$

$$F(y) = P\int dx\ \frac{x}{x-y}\ e^{-x^2/2} \tag{2.24}$$

Then

$$\mathrm{Re}\ X(q) = \frac{\gamma\omega_p^2}{\sqrt{2\pi}\ (kv_{th})^2}\ F(y) \tag{2.25}$$

We can rewrite Equation (2.24) as

$$F(y) = \sqrt{2\pi} - \pi y\ \mathrm{Im}\ w(y/\sqrt{2}) \tag{2.26}$$

where,

$$w(z) = \frac{i}{\pi}\int_{-\infty}^{\infty} \frac{e^{-t^2}\,dt}{z-t} \tag{2.27}$$

Thus, $F > 0$ for small y, and changes sign at some y of order 1. We are interested in the largest negative value of F. It occurs for $y \approx 2.1$, and its value is $\approx -0.71 \approx -1/\sqrt{2}$

Thus, $$F_{min} \approx -1/\sqrt{2} \tag{2.28}$$

Equation (2.18) gives

$$\omega_p^2 = \frac{2\sqrt{\pi}}{\gamma} (kv_{th})^2 \tag{2.29}$$

Using the approximation again,

$$k \approx k_s \approx 2\gamma^2 k_o \tag{2.30}$$

Equation (2.29) becomes

$$\omega_p^2 = \sqrt{2\pi}\ \gamma^3 (kv_{th})^2 \tag{2.31}$$

As a numerical example, consider $10.6\,\mu$ laser as a pump. Let $\gamma = 2.3$, $k_BT = 0.1$ eV, we find

$$J \sim 6 \times 10^6 \text{ amp/cm}^2 \tag{2.32}$$

An extremely high electron current! This same amount of current density is also required for a high voltage FEL with $\gamma = 42.4$, $E = k_BT = 10^3$ eV, and wiggler wavelength 4.2 cm.

Nevertheless, for low voltage FEL with $100\,\mu$ radiation as a pump, the required current density for $\gamma = 2.3$, $k_BT = 0.1$ eV becomes

$$J = 6 \times 10^4 \text{ amp/cm}^2 \tag{2.33}$$

which is an achievable current density.

2.3 Physical Considerations

As mentioned earlier, the simple gain expressions obtained in Section 2.2 are, among other things, based on the condition that $k_o r_\perp \ll 1$. If this is violated, then the one-dimensional approximation neglecting the transverse variation of the pump is not justified. This kind of effect was examined in an earlier work for high voltage FEL[20]. A quantitative assessment of this effect is still in progress.

The theory of transverse gradient was developed by approximating the electron trajectory under the condition $\sqrt{P}/\gamma \ll 1$. The analytical approximation is tested for $P = 1$ and $\gamma = 100$. The accuracy is seen to be within 2% or so[20]. A preliminary numerical calculation on the extraction efficiency seems to indicate that the transverse gradients lead to no significant detrimental effect.

For $P \approx 1$, our analytical approximations for the electron trajectories break down for low voltage FEL. The electron trajectories under these conditions are expected to be rather involved. This is due to the fact that the betatron oscillation period decreases rapidly with decreasing γ. This oscillation is in part due to nonvanishing axial helical magnetic field when the electron moves off the axis. Furthermore, the quadratic nonlinearity due to the transverse helical magnetic fields produces higher Fourier components in the electron trajectories. These effects on energy extraction should be further investigated not only in the single-particle limit, but also in the collective regime, which is expected to be important for low voltage FEL.

The transverse effects are expected to be important only for the first stage involving magnetostatic wiggler. With an electromagnetic pump, there is very little axial magnetic field to produce significant betatron oscillations on the electron beam. Furthermore, inhomogeneity of the pump field in the transverse plane has negligible effect on the resonance condition with an electromagnetic pump of wavelengths less than 1 mm.

It was pointed out that low voltage FEL should operate in the high electron current regime in order to produce high power radiation. When the current density becomes high, besides the collective effect in the mean field approximation described in Section 2.2, the Boltzmann collision integral can not be ignored. A possible consequence due to binary collisions involving two electrons is the quick thermalization of electrons in the beam frame. We now determine the relaxation time that the temperature of transverse and longitudinal motion will equalize in the beam frame.

The cross section for coulomb collisions involving two electrons is, in the center of mass system,

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{2\mu v^2}\right)^2 \left[\frac{1}{\sin^4\frac{\theta}{2}} + \frac{1}{\cos^4\frac{\theta}{2}} - \frac{1}{\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}} \cos\left(\frac{e^2}{\hbar v}\ln\tan^2\frac{\theta}{2}\right)\right] \quad (2.34)$$

where $\mu = m/2$, v is the relative velocity and

$$d\Omega = 2\pi \sin\theta \, d\theta \quad (2.35)$$

Multiplying by $1 - \cos\theta$ and integrating from $\theta = 0$ to $\Pi/2$ gives the following transport cross section[21],

$$\sigma = 2\pi \left(\frac{e^2}{\mu v^2}\right)^2 \text{Log}\,\tilde{\Lambda} \quad (2.36)$$

where Log $\hat{\Lambda}$ is the usual logarithm involving screening; we take it to be 5.

The fundamental process leading to Equation (2.34) is shown in Figure 2.

The relative kinetic energy has the mean value

$$\tfrac{1}{2}\mu\langle v^2\rangle = \frac{3}{2} k_B T \qquad (2.37)$$

The mean free path is, with n denoting the electron density,

$$\ell = (n\sigma)^{-1} \qquad (2.38)$$

The time between collisions in the beam frame is

$$\tau = \ell/v = (n\sigma v)^{-1} \qquad (2.39)$$

and the electron beam becomes essentially random in direction of motion at a distance Λ,

$$\Lambda = c\beta\gamma\tau \qquad (2.40)$$

For $k_B T = 0.1$ eV, $v = 3\times10^7$ cm/sec. For electron density given by Equation (2.33), the mean free path is

$$\ell = 10^{-2} \text{ cm} \qquad (2.41)$$

and collision time is

$$\tau = 3 \times 10^{-10} \text{ sec} \qquad (2.42)$$

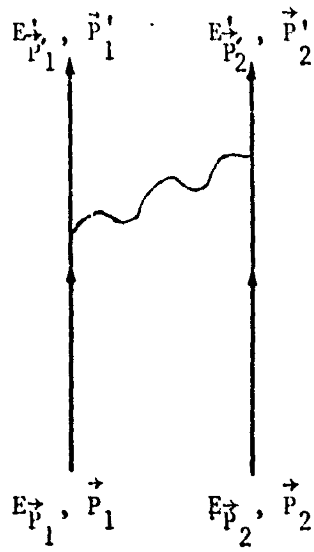

Figure 2. Electron - Electron Scattering. The Exchange Term Arises from the Antisymmetrization of the Two-Electron States Vector according to the Pauli Exclusion Principle

The randomization distance is

$$\Lambda = 20 \text{ cm} \tag{2.43}$$

for $\gamma = 2.3$. Note that this distance is smaller than the typical interaction length, which is a few meters in most designs.

The physical process described above can be incorporated into the Boltzmann equation (the collision integral). This should be included in the calculation of dispersion relation rather than just the self-consistent field alone as in Section 2.2. As a first cut to estimate its effect, it seems to be reasonable to use a relaxation time approximation so that the theory can be greatly simplified.

2.4 Other Relevant Work

In this section, we shall briefly describe two relevant papers that seem to be more general in their approach. The first is a self-consistent version description of the FEL instability developed by Davidson and Uhm[22]. The second is a unified theory of different forms of the free electron laser such as Cerenkov-Smith-Purcell laser, Compton FEL and Raman FEL[23].

The theory of Davidson and Uhm[22] starts with the Maxwell-Vlasov equations, as usual. The analysis assumes a relativistic electron beam with uniform cross section propagating in the Z direction. It further assumes that the beam current density is sufficiently small. This latter assumption enables one to neglect the equilibrium space charge effects.

For the purpose of stability analysis and derivation of a dispersion relation, the coupled Maxwell-Vlasov nonlinear equations were linearized, as was done in the past by most investigators. Instead of using resonant approximation by considering a single frequency of the radiation field, these authors expanded the potential perturbation and the perturbed electron distribution function in Fourier series in terms of arbitrary harmonic number of the fundamental wiggler wave number k_o. A detailed analysis of the linearized Maxwell-Vlasov equations then leads to a matrix dispersion relation containing a coupling of adjacent harmonic numbers by the degree of off-diagonality in the matrix elements. This is more general than the usual dispersion where off-diagonal matrix elements vanish. In this approach, Davidson and Uhm took the initial electron beam distribution without transverse momentum spread.

In order to examine the consequence of this new approach, Davidson and Uhm then considered the case of a cold electron beam without longitudinal momentum spread. They showed that for weak pump, the small signal gain was similar to the conventional results. The peak gain occurs near the harmonic number n satisfying the back scattered Doppler shift expression, namely, $n \approx 2\gamma^2$ where γmc^2 is the electron energy. The effect of off-diagnonal coupling under the weak pumping condition is seen to be negligible. However, for strong pump and strong beam current where the beam plasma frequency ω_p is comparable to $k_o c$, substantial corrections to the gain bandwidth are found as compared to the theory of Kroll, etc.

Nevertheless, when the strong pump and strong beam current are considered, the analysis based on the small signal gain loses its practical importance, because here one is interested in the nonlinear saturation phenomena, and the linearization approximation cannot be used. It is of great interest to see how the method of Davidson and Uhm can be applied to the study of the nonlinear saturation effect. This has not been done, but one can no doubt expect considerable mathematical complications. A computer study is probably rather expensive.

Based on the linearization approximation, Davidson and Uhm also estimated the influence of beam thermal effects on stability properties. An expression was derived to estimate the energy spread required for heavy Landau damping of the fundamental longitudinal mode. A numerical example is given for $\gamma = 5$, $\omega_p/k_0c = 0.71$. The fractional energy spread is found to be less than 7% in order for the FEL to have a gain.

The method of Davidson and Uhm seems to be quite general. The useful results, however, can also be derived using a simpler approach as was done by Kroll. It is not clear how this method can be generalized to account for the transverse gradient and transverse momentum spread effects. The algebra is probably prohibitive. Further elaboration of this approach does not seem to be practical.

We now discuss the report by Gover and Sprangle[23]. This article describes in a comparative way the main operating parameters of various FELs. It provides a useful tool for laser design and comparative evaluation of the various lasers. The authors show that the various kinds of FELs satisfy the same gain-dispersion relation. The difference consists of only a single coupling parameter. The authors determined the small signal gain in all the gain regimes. The laser gain parameter, radiation extraction efficiency, maximum power generation and spectral width were given and compared in the various kinds of FELs and gain regimes. Some expressions derived earlier for the magnetic bremsstrahlung FEL with space charge effect corrections were shown to be special cases of more general expressions given in this report.

The possibility of achieving a unified theory of magnetic bremsstrahlung, electrostatic bremsstrahlung, Compton-Raman scattering and Cerenkov-Smith-Purcell FELs is based on the principle which requires energy-momentum conservations in an elementary process. The Doppler effect can be shown to be a special case of the conservation laws under the condition that the photon energy is much less than the electron rest energy. The coupling parameters for various FELs considered are determined by the momentum transfer integral. Thus, it is not surprising to obtain a unified dispersion relation if the coupling parameters are evaluated from the lowest order Born approximation. It is of interest to observe that the coupling parameters for Cerenkov, Smith-Purcell and longitudinal electrostatic FELs are all proportional to the scattered wavelength, while for transverse electrostatic, magnetic bremsstrahlung and Compton-Raman lasers, they are inversely proportional to the scattered wavelength. Moreover, only magnetic bremsstrahlung and Compton-Raman lasers are of interest to ONR low voltage FEL.

An alternative view on the origin of the similarity of the various FELs can be stated in terms of those used in the plasma physics. According to Gover and Sprangle, the various FELs considered here all involve longitudinal coupling between single electrons or electron plasma waves and an electromagnetic wave. It is obviously so for the Cerenkov-Smith-Purcell FELs, but also in the magnetic bremsstrahlung FEL in which the electromagnetic wave has a transverse field only, and the electron beam is transversely modulated by the static magnetic field. Thus, there is a longitudinal interaction between the electromagnetic wave and electron beam plasma, carried out through the ponderomotive potential (radiation pressure).

The derivation of dispersion relation by Gover and Sprangle used the method of the Laplace transform, rather than the Fourier expansion as used by Davidson and Uhm discussed earlier and many others. The results obtained are, of course, the same. The small signal gain can be obtained by examining the inverse Laplace transform of the signal amplitude using the one-pole approximation when the inverse integral is evaluated according to the Cauchy theorem.

This report also contains a discussion of maximum power as a function of wavelength. It is shown that in the high gain warm beam case, there is a dependence like λ^{13} in the bremsstrahlung FELs, and λ^{12} in the Compton-Raman FEL. This is indeed particularly encouraging for the ONR program. Unfortunately, there is no experimental evidence to support these predictions.

SECTION 3

3.0 PARTICLE TRAPING AND HIGH EXTRACTION

3.1 Introduction

It has been shown that the constant parameter magnetostatic wiggler can result in an energy extraction of ~ 1/2N per pass where N is the number of periods provided the FEL is homogeneously broadened. For the case of inhomogeneous broadening, the extraction efficiency is even lower. To improve the extraction efficiency in a single pass, it was proposed to vary the parameters of the wiggler in such a way that the ponderomotive potential well in which the electrons move is decelerated and, thereby, decelerating the electrons. In a magnetostatic wiggler, both the period, as well as the wiggler magnetic vector potential, can be designed to vary in a prescribed manner. However, with an electromagnetic pump of wavelengths less than 1 mm, first, the pump vector potential is too small to contribute significantly to the resonance condition; and secondly, it is not possible to change the wavelength of the pump. It has been suggested that the electron beam can be made to interact at varying interaction angle to the pump radiation (instead of π radians) such that the effective pump wavelength is altered. Such a scheme, though attractive, would severely restrict the interaction length. An alternate scheme is to use an axial accelerating field to keep the electrons at resonant energy and to keep them trapped in the ponderomotive potential well. For effective trapping, the electrostatic coulomb force between the electrons has to be much smaller than the attractive ponderomotive potential force. In this regime of operation, then single particle physics is applicable.

3.2 Scaling Relations

If the electrons are trapped in the ponderomotive potential well and kept in resonance by an accelerating electric field while the electrons are slowed by the ponderomotive potential, the electric field necessary for acceleration is given by

$$E_{acc} = \frac{mc^2}{e} \quad \frac{a_p e_s \sin\psi_r}{\gamma_r} \tag{3.1}$$

In the low voltage FEL, we anticipate the total energy spread to be larger than the bucket height. Under these circumstances, the optimum phase angle for trapping is $\sim 24.5^{\circ}$. In Equation (3.1), a_p is the normalized vector potential of the pump ($= eE_p \lambda_p/2\pi\, mc^2$) and e_s is the normalized scattered field ($e_s = eE_s/mc^2$).

Assuming the pump to be an electromagnetic wave and that $4\gamma^2 \approx \lambda_p/\lambda_s$, Equation (3.1) can be written as

$$E_{acc} = 9.75 \times 10^{-9} \left[I_s I_p \lambda_{p\mu} \lambda_{s\mu} \right]^{\frac{1}{2}} , \text{ v/cm} \tag{3.2}$$

where I_p and I_s are intensities in w/cm^2 and $\lambda_{p\mu}$ and $\lambda_{s\mu}$ are the pump and output wavelengths in microns.

The capture efficiency for trapped electrons is given by

$$\eta_c = \frac{J/\gamma}{2\pi(\Delta\gamma/\gamma)_T} \tag{3.3}$$

where J is the area of the bucket in phase space. From Jason Report 24 No. 79-01, p. 29, we find

$$J = \frac{16\gamma}{\mu} \sqrt{a_s a_p} \quad \alpha\,(\Psi_r) \tag{3.4}$$

where $\mu^2 = 1 + a_p^2 + a_s^2 \simeq 1$ for low voltage electromagnetic pump.

$$a_p = 6.051 \times 10^{-10} \sqrt{I_p \cdot \lambda^2_{p\mu}} \tag{3.5}$$

and $\alpha\,(24.5^o) \approx 0.4$.

The energy spread $(\Delta\gamma / \gamma)_T$ consists of three parts. The first one is due to the natural energy spread. This could be very small, i.e., less than 10^{-4}. The second is due to coulomb potential in the beam. This is approximately 30 volts/ampere of current. This could be substantial leading to a spread in the longitudinal gamma. However, using Brillouin flow, one can make the beam with constant axial gamma. We shall assume that this is done. The third contribution to the spread comes from the emittance of the beam. It has been shown by Kelvin Neil[25] that the spread, due to emittance, is given by

$$\left(\frac{\Delta\gamma}{\gamma}\right)_\varepsilon = \frac{10^{-5}\,\pi\alpha^2}{2} \quad J_{eb} \tag{3.6}$$

where J_{eb} is the current density in amp/cm^2 and α is a number between 1 and 5 and is usually taken to be equal to 2 for good beams using thermionic guns. This being the dominant spread mechanism for the axial gamma, we may write the capture efficiency as

$$\eta_c = 3.92 \times 10^{-5} \left[I_p I_s \lambda_{p\mu}^2 \ \lambda_{s\mu}^2 \right]^{1/4} \ \frac{1}{\alpha^2 J_{eb}} \tag{3.7}$$

Let C_c be the optical output coupling coefficient. The extracted flux, by definition, is equal to $I_s C_c$. If L is the interaction length in the FEL, the extracted flux is also equal to

$$I_{ext} = E_{acc} \ L \ J_{eb} \ \eta_c \tag{3.8a}$$

$$= 3.82 \times 10^{-13} \ (I_p I_s)^{3/4} \ \frac{\lambda_{p\mu} \lambda_{s\mu} \ L}{\alpha^2} \tag{3.8b}$$

which is independent of the current density. This is not surprising, since the capture efficiency decreases inversely as the current density. Equating I_{ext} and $I_s C_c$, we obtain the steady-state cavity flux as,

$$I_s = 2.14 \times 10^{-50} \ I_p^3 \ \lambda_{p\mu}^4 \ \lambda_{s\mu}^4 \ \frac{L^4}{\alpha^8 C_c^4} \tag{3.9}$$

The interaction length L is usually limited by the diffraction of the pump beam. We can take the product $I_p \lambda_{p\mu} L$ to be equal to $4 \times 10^4 P_p$ where the factor 10^4 is used in converting cm to microns and P_p is the pump power in watts; we can write Equation (3.9) as

$$I_s = 1.37 \times 10^{-36} \ \frac{P_{p\mu}^3 \lambda_{p\mu} \lambda_{s\mu}^4 L}{\alpha^8 C_c^4} \tag{3.10}$$

The important point to note from Equation (3.10) is that a factor of 2 improvement in the emittance (reduction of α by a factor of 2) results in an increase of cavity flux by 256 and, hence, also the extracted power and efficiency. This eighth power dependence on emittance has also been obtained by Kelvin Neil. In Figure (3), we have plotted I_s versus pump power for $\lambda_{p\mu} = 400\mu$, $\lambda_{s\mu} = 0.5\mu$, $C_c = 10^{-2}$ and $L = 100$ cm for three different values of α. In Figure 4, we have plotted I_s versus pump power for three different pump wavelengths using $\alpha = 2$, $\lambda_{s\mu} = 0.5\mu$, $L = 100$ cm and $C_c = 10^{-2}$. The figure illustrates the difficulty in increasing the cavity flux as the pump wavelength is reduced.

Neglecting the energy supplied by the accelerating field, the extraction efficiency is given by

$$\eta_{ext} = \frac{I_{ext}}{(\gamma - 1)\, mc^2\, J_{eb}} \tag{3.11a}$$

$$= \frac{2.68 \times 10^{-42}\; P_p^3\; \lambda_{p\mu}\; \lambda_{s\mu}^4\; L}{(\gamma - 1)\; \alpha^8\; C_c^3\; J_{eb}} \tag{3.11b}$$

In Figure (5), the extraction efficiency is plotted as a function of pump power for various values of the parameter α while fixing the equivalent spread due to emittance at 0.1%. This meant fixing $\alpha^2 j_{eb}$ at $\sim$ 64 amp/cm^2. The figure also points out the important gain that can be achieved in improving the beam emittance. The horizontal hatched line indicates a trapped fraction of 40% which is about the maximum that one can achieve in a nonadiabatic accelerating field. This is also the cold beam limit. The present calculation would, therefore, be invalid above the hatched line.

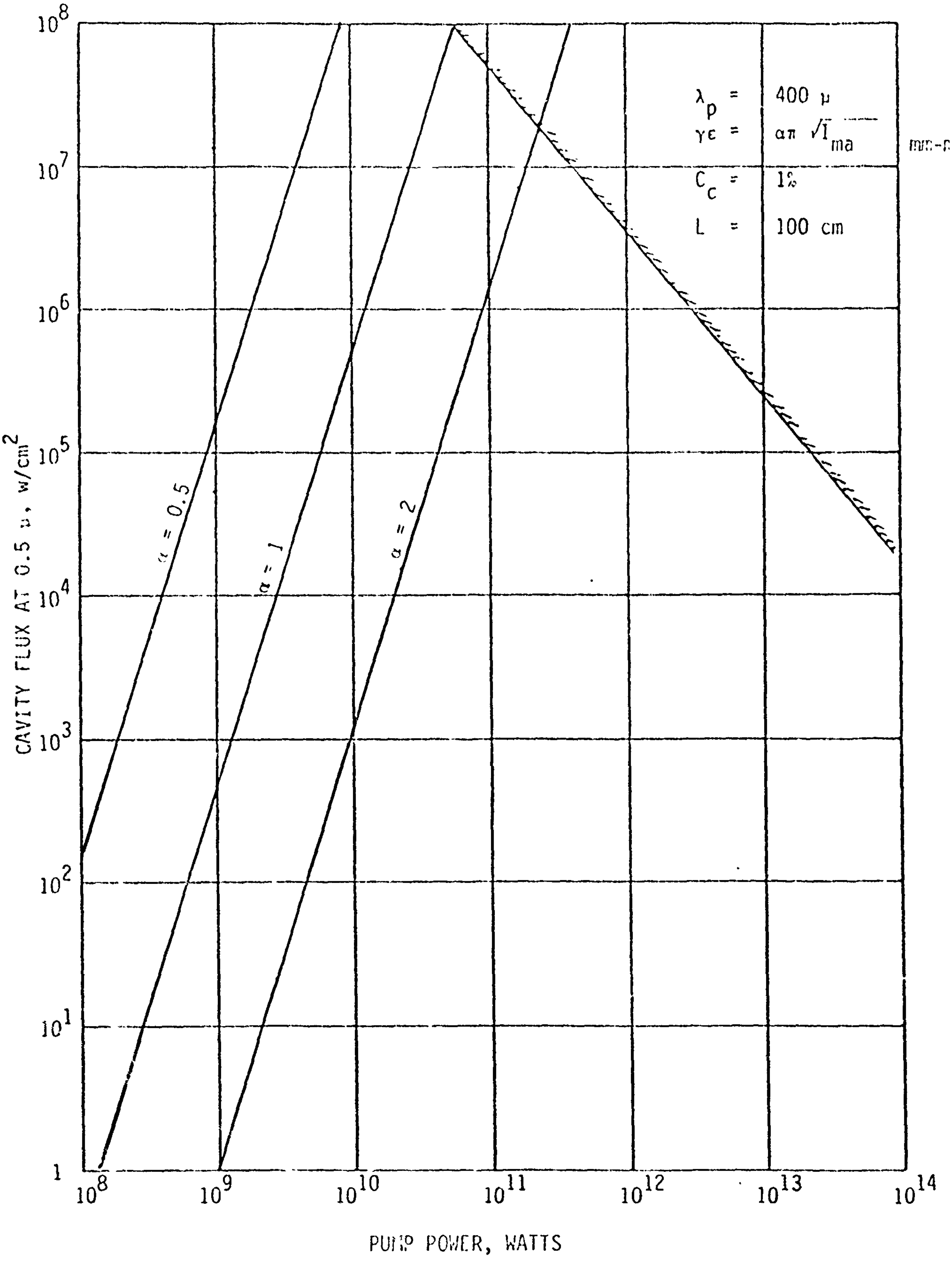

Figure 3. Effect of Emittance on Cavity Flux for Trapping

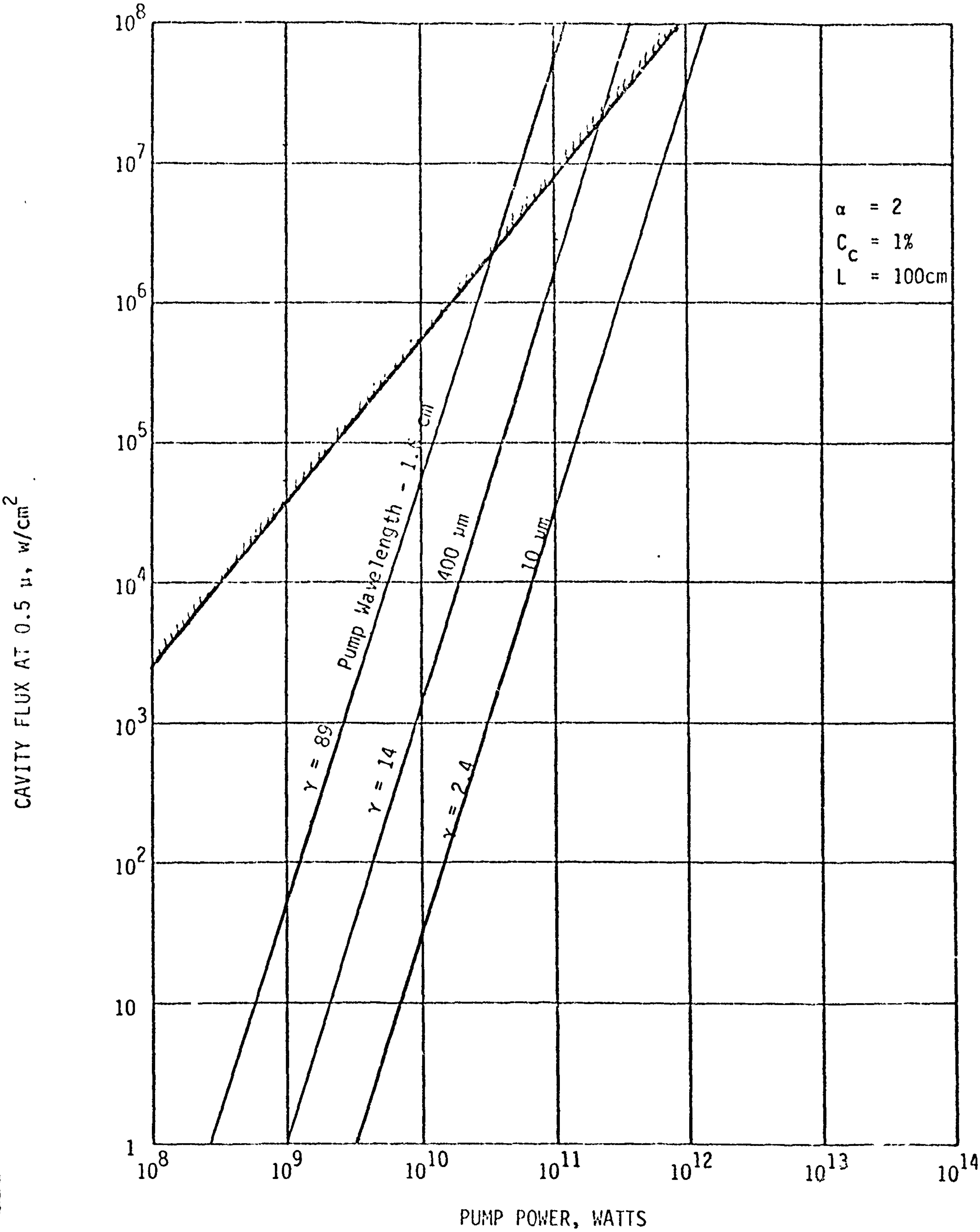

Figure 4. Effect of Pump Wavelength on Cavity Flux

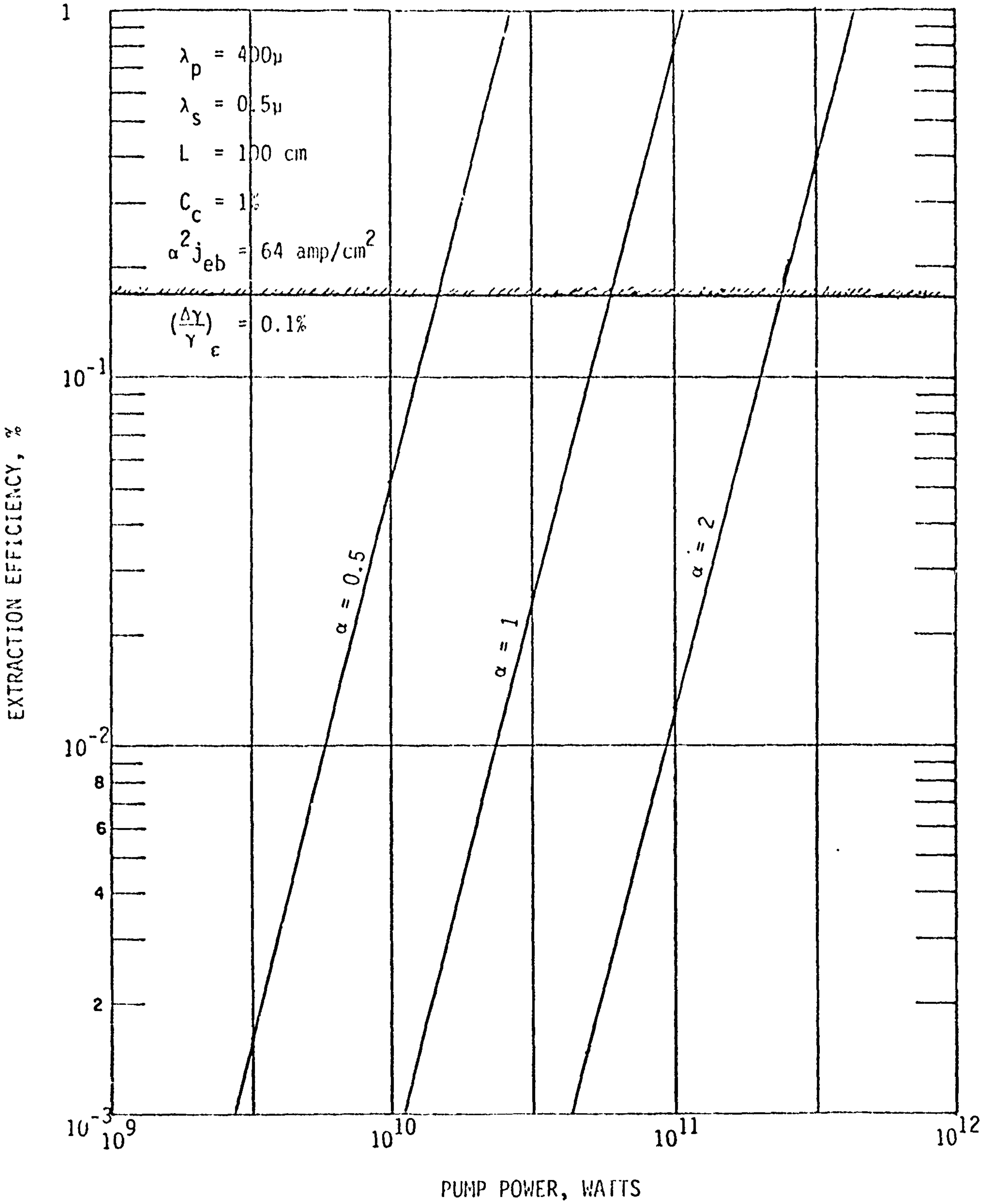

Figure 5. Effect of Emittance on Extraction Efficiency

SECTION 4

4.0 FEL OPTICS

4.1 Introduction

The two stage free electron laser, as we have seen in the previous section, has an extremely low single pass efficiency, and the fraction of pump photons that are back-scattered by the electron beam and up converted in the laboratory frame is also very low. It, therefore, becomes imperative that one make a very high Q cavity at the pump wavelength so that one may re-circulate the pump photons many, many times. This, in turn, would reduce the energy cost of producing the pump photons. As we shall see presently, the pump cavity should be designed with a loss of 10^{-3} or less per round trip. Secondly, the extraction efficiency improves dramatically with increased pump power and signal power at the output wavelength. In order to improve the extraction efficiency, one must operate the low voltage free electron laser at a very low output coupling. Thus, the cavity losses at the output wavelength (including the output coupling) should typically be limited to 1% or less. Thirdly, the high cavity fluxes necessitate long cavity lengths or advanced optical designs to mitigate damages to the optical surfaces. These are discussed below.

4.2 Technical Issues

The overall system efficiency of a low voltage free electron laser that utilizes an electromagnetic pump can be written as

$$\eta_{sys} = \frac{\eta_c\, E_{acc}\, L\, I}{(1-\eta_R)\; I\left[(\gamma-1)mc^2 + (1-\eta_c)\, E_{acc} L\right] + \eta_c\, E_{acc}\, L\, I + \beta\, P_p/\eta_p} \qquad (4.1)$$

where η_R is the recovery efficiency of the electron beam energy, β is the round trip fractional loss of the pump beam and η_p is the efficiency of the pump laser system. In Figure 6, we have plotted the system efficiency versus β for a representative set of parameters with P_p as the third variable. It is clear from the figure that for devices of interest to Navy (medium to high powers) that the round trip losses of the pump beam must be less than 10^{-3} for any reasonable system efficiency. We are thus forced to design reflective optics for the pump cavity with > 99.9% reflectance per surface.

For output powers in excess of tens of kilowatts in the visible/near IR, the circulating pump power required is in excess of 10^{10} watts, while the circulating power at the short wavelength would be in excess of 10^6 watts. One is then faced with the problem of designing and developing optical cavities at two disparate wavelengths with high to very high reflectivity at both the wavelengths. The optical design could be somewhat simplified if means can be found for separating the two wavelengths so that one can concentrate on the design of high reflectivity optics at a single wavelength. Several methods have been suggested for separating the pump and output optical beams. These are discussed in the next section.

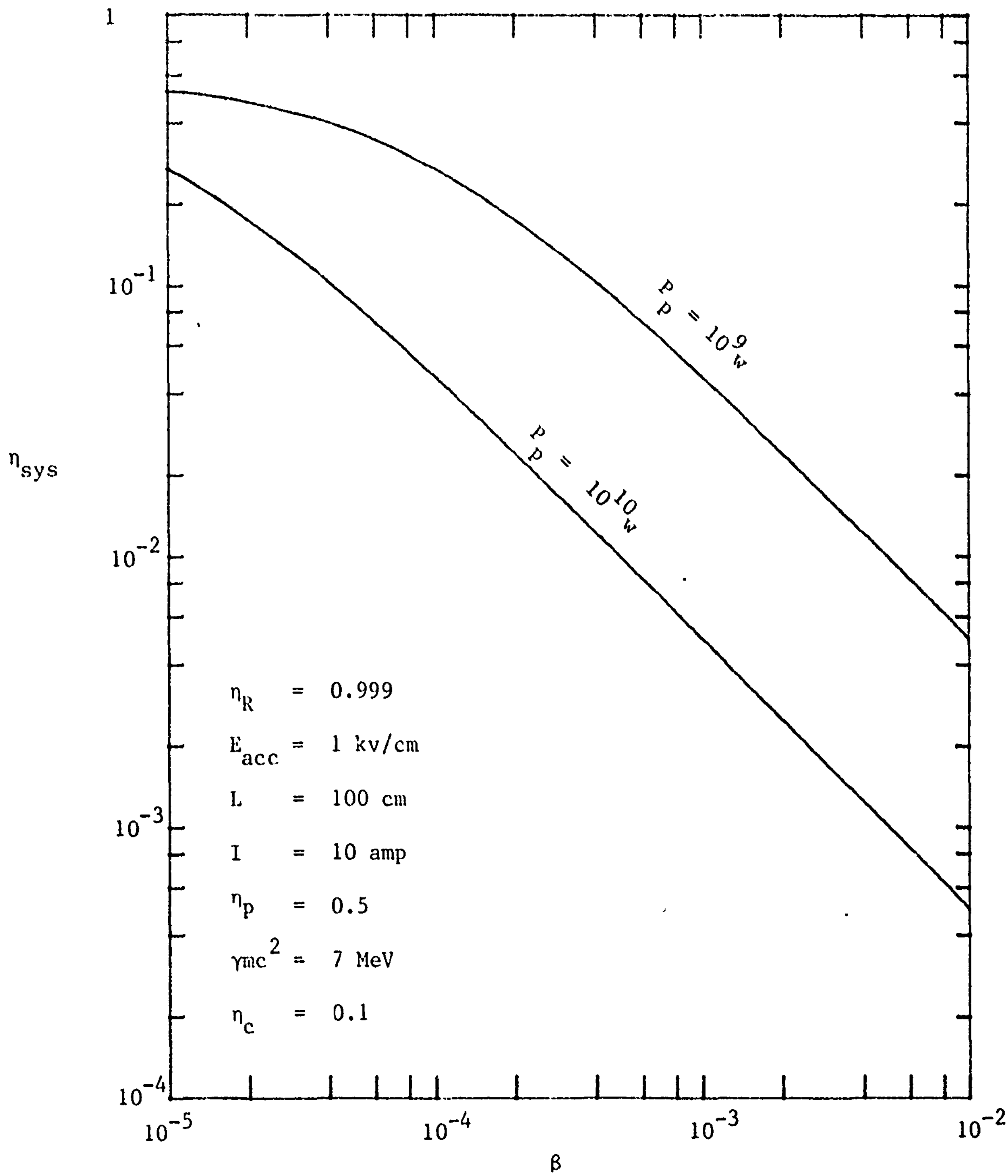

Figure 6. System Efficiency vs. Round Trip Pump Cavity Loss

4.3 Technical Analysis

The three suggested solutions for separating the pump and output beams are (1) use of diffraction grating, (2) non-collinear propagation and (3) use of diffraction spreading. In the first method, a grating is used to diffract the pump and output beams in different directions. Normally, this is an excellent method to separate radiation at two different wavelengths. However, it seems that the introduction of any grating would produce losses exceeding 10^{-3} per round trip, and thus the method would have to be ruled out on the basis of this.

The second scheme consists in having the pump and output beam interact at a non-zero angle as shown in Figure 7. However, because of the constant diffraction spreading of the pump beam, the intersection angle has to be greater than the diffraction angle. This would limit the interaction distance. For example, with a pump beam of one centimeter diameter at 500 microns, the diffraction angle θ is approximately 61 milliradians ($1.22\lambda_p/d$). The interaction length for an interaction angle of θ is $d/\sin\theta \sim d/\theta = d^2/1.22\,\lambda_p \sim$ 16 cm. To increase the interaction length, one has to increase d or decrease λ_p, both of which would increase the pump power requirements. In general, therefore, this is not a satisfactory solution for separating the two beams.

The third scheme takes advantage of the difference in the diffraction spreading of the pump and output waves to separate the two beams. If the beam waist diameters are equal at the two wavelengths,

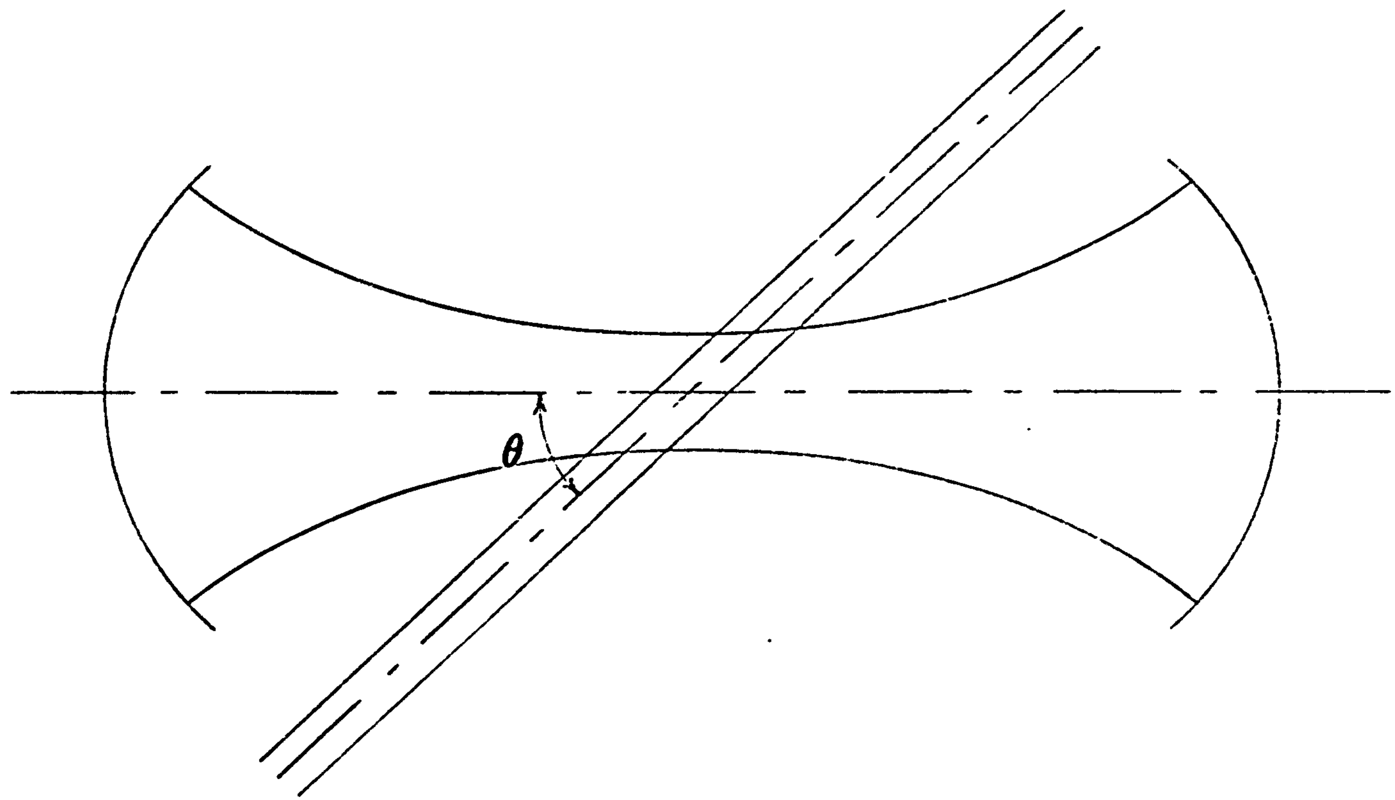

Figure 7. Schematic of Interaction at an Angle for Separating the Pump and Output Beams

then the pump beam spreads out at an angle approximately $4\gamma^2$ larger than the output beam. This is just the ratio of the two wavelengths under consideration. Figure 8 shows a sketch of the scheme. The small hole in the pump cavity through which the short wavelength is decoupled from the pump represents a loss to the pump beam. If the pump intensity on the mirror peaks on axis, the hole can be shifted slightly off-axis to reduce pump beam losses. The following simple calculation gives how much pump power is lost because of the hole in the center of the pump cavity. If L_w is the interaction length (Rayleigh range of the pump beam), the pump beam waist radius is given by $w_0 = (\lambda_p L_w/2\pi)^{\frac{1}{2}}$. Let L_c be the mirror separation distance in the pump cavity. The 1/e radius of the pump beam at the mirror is then given by

$$\begin{aligned} w_p &= w_o \left[1 + (\lambda_p L_c/2\pi w_o^2)^2\right]^{\frac{1}{2}} \\ &= w_o \left[1 + (L_c/L_w)^2\right]^{\frac{1}{2}} \end{aligned} \tag{4.2}$$

while the radius of the output beam is given by

$$w_s = w_o \left[1 + (L_c\lambda_s/L_w\lambda_p)^2\right]^{\frac{1}{2}} \tag{4.3}$$

If the radius of the central hole is equal to w_s, fractional loss of pump power because of the hole is given by

$$\begin{aligned} f &= 2w_s^2/w_p^2 \\ &= \frac{2\left[1 + (L_c\lambda_s/L_w\lambda_p)^2\right]}{\left[1 + (L_c/L_w)^2\right]} \end{aligned} \tag{4.4}$$

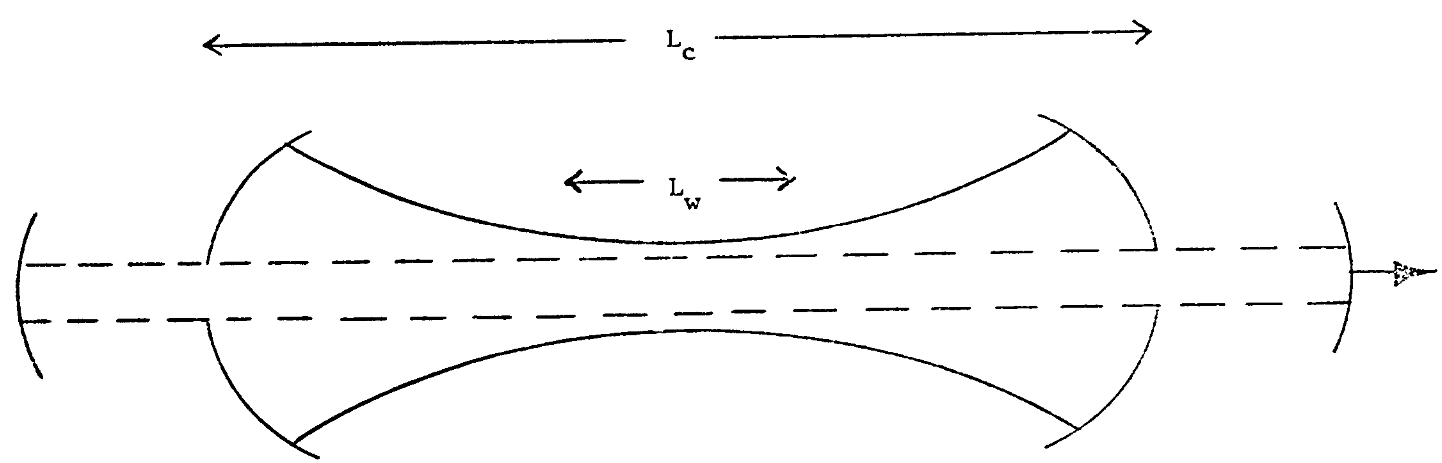

Figure 8. Schematic of Separating Pump and Output Beams by making use of Diffraction Spreading of the Pump Beam

where the factor 2 arises because of the losses at both the mirrors. In Figure 9, we have plotted the fractional loss f versus (L_c/L_w) for different ratios of the pump and output wavelengths. The plot shows that the losses can be minimized by operating the FEL at longer pump wavelengths. This is consistent with the conclusions of the previous section wherein we found that the extraction efficiency and output power improved with larger pump wavelength. The actual cavity loss β at the pump wavelength is the sum of the loss calculated above and absorption losses at the mirrors.

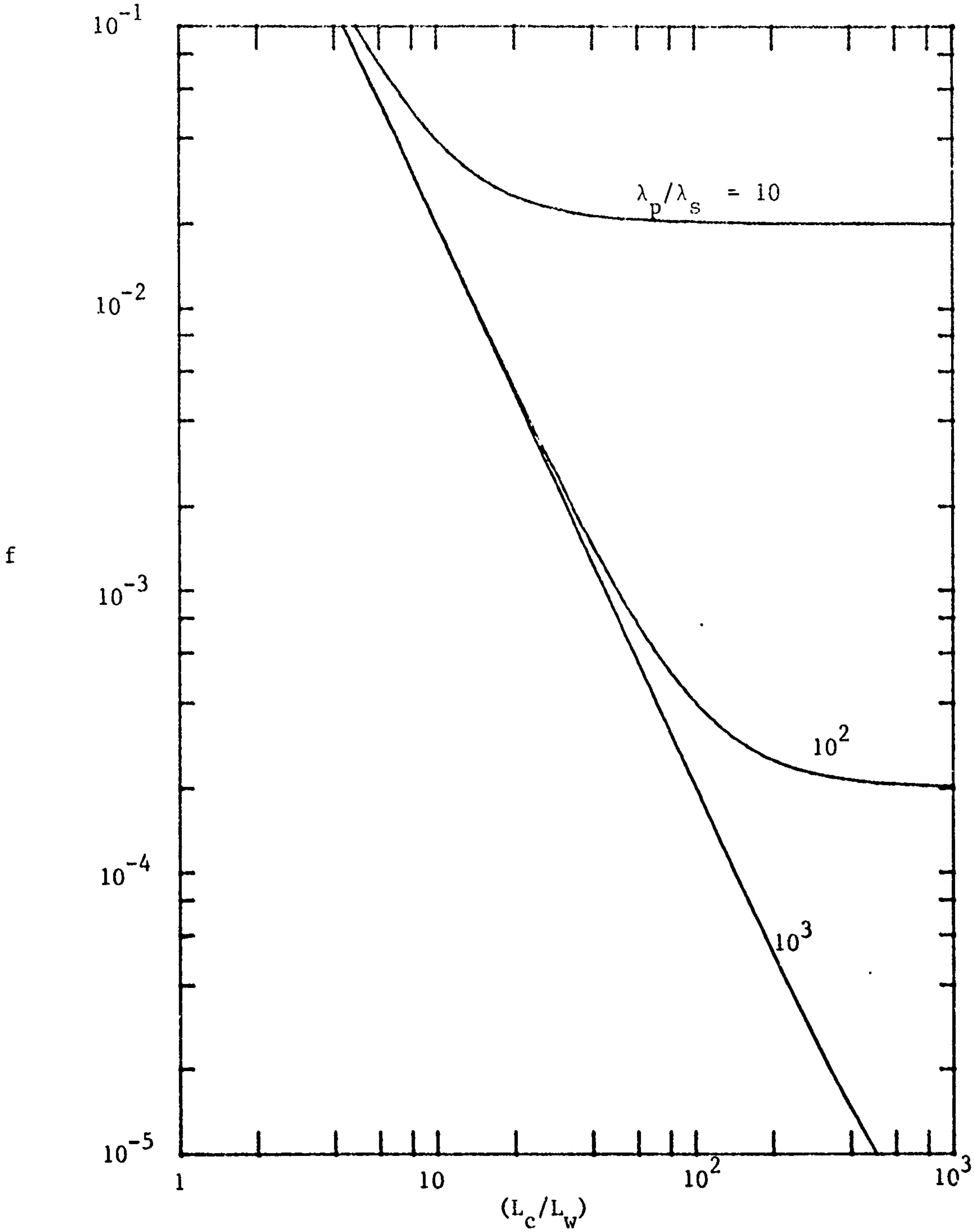

Figure 9. Fractional Pump Power Loss vs. (L_c/L_w) for Different Ratios of Pump and Output Wavelengths

SECTION 5

5.0 SUMMARY

We have surveyed the theory of the low voltage FEL. The most comprehensive theory of small signal gain of the free electron laser has been given by Kroll and McMullin[18]. Their results contain both single particle and collective effects. Their results have been summarized and mapped conveniently by us so that one can immediately obtain small signal gain for any set of given pump and electron beam parameters. The map covers all regimes of interest, i.e., single particle regime, collective effect regime and includes both warm and cold beam limits.

We have determined that in the interesting regime of "high" extraction, single particle physics dominates over collective effects. This condition is achieved by keeping the particles trapped in the ponderomotive potential in resonance (by varying the wiggler parameters in the case of magnetostatic wiggler and by re-acceleration with axial electric field in the case of the electromagnetic pump). We have, therefore, been able to utilize the theoretical analyses carried out for the variable parameter wiggler and have been able to calculate the axial electric field necessary for acceleration, the maximum energy spread that can be allowed without loss of trapping efficiency, the requirement on the emittance of the electron beam and the extraction efficiency that can be obtained. For an oscillator, we have been able to calculate the maximum output flux and power that one can achieve with the given pump parameters.Our preliminary conclusion is that the electron beam emittance has to be improved by a factor of ~ 4 for more efficient FEL operation. We have also found that the pump parameter requirements become less stressing at larger pump wavelengths. Thus, one would prefer a 100μ pump wave rather than 10μ pump that is obtained from efficient CO_2 lasers.

An important technical issue is the feasibility of constructing optical cavities at the pump and output wavelengths. The reason for recirculating the pump flux is that only a small fraction of the pump photons get "used" (scattered back by the electron beam into high frequency photons). Overall system efficiency is determined by the efficiency of the generation of the pump photons and the efficiency of the use of pump photons for up conversion. Typically, one would require fractional pump energy losses in the pump cavity to be less than 10^{-3} per pass or even 10^{-4}. The higher the Q of the pump cavity, the less stringent are the requirements on the electron beam that generates the pump photons. Because of the low gain calculated for the second stage of this FEL concept (for reasonable pump parameters), one would design a cavity for the second stage with low output coupling and high Q. Since, in general, it is difficult to make a single high Q cavity at two such widely different wavelengths, we have investigated schemes for separating the pump and output radiation. The simplest and most practical scheme seems to be one where one uses the diffraction differences of the two beams to separate them. It is found that for this scheme to work efficiently, one requires a large ratio between pump and output wavelengths, thus favoring 1000μ to 100μ pump wavelengths for an output wavelength of $0.5\ \mu$. This choice is consistent with that based on extraction efficiency and FEL physics.

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$$P = 8.1 \times 10^{-8} \, (\lambda_o / 10.6\mu)^2 \quad (I_o/10^9 \text{ w/cm}^2)$$

$P \approx 1$ for Wiggler pump, but $P << 1$ in general for electromagnetic pump.

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